

THEORY OF DARK ENERGY AS A SELF-REPLICATING FLUID IN A CLOSED UNIVERSE: GRAVITATIONAL COMBUSTION, BLACK-HOLE EVAPORATION AND THE MASTER SCALAR-TENSOR LAGRANGIAN

A unified framework in which matter and black holes dissipate
into a self-replicating dark-energy fluid that inflates the cosmic balloon
at an ever-increasing rate

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Abstract. A reformulated theoretical framework is presented in which dark energy is identified with a real scalar field ϕ behaving as a *self-replicating fluid* confined within the closed three-geometry of the Universe, in direct analogy to water sealed inside an inflating balloon. The fluid is produced by two irreversible channels: (i) *gravitational combustion* of ordinary matter in strong curvature regions, and (ii) *enhanced Hawking evaporation* of black holes. Once the local amplitude of ϕ exceeds a critical threshold ϕ_c , a tachyonic instability activates a quantum self-replication process at rate $\Gamma_{\text{rep}} = |m_{\text{eff}}^2|/(4\pi)$, so that the fluid amplifies itself in addition to being fed by the dissipating matter sector. As the Universe cools and matter together with the gravitating black-hole population decay into ϕ , the closed cosmic balloon is forced to expand at an ever-increasing rate. The theory is built on five postulates (P1–P5) and encoded in a complete action $\mathcal{S} = \int \sqrt{-g} \mathcal{L} d^4x$ with Lagrangian $\mathcal{L} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{GHY}} + \mathcal{L}_{\phi} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{burn}} + \mathcal{L}_{\text{evap}} + \mathcal{L}_{\text{rep}} + \mathcal{L}_{\text{mat}}$. The modified Einstein–Klein–Gordon system is derived rigorously from the variational principle, the modified Friedmann equations for the closed FLRW geometry ($k = +1$) are obtained, and the coupled continuity equations for ρ_m , ρ_{BH} and ρ_{ϕ} are shown to admit a late-time asymptotic solution $a(t) \sim \exp[\exp(\Gamma_{\text{rep}} t/2)]$ *without* violating the weak energy condition. The complete microphysics (quasiparticle spectrum, self-replication cross section, mean free time, coherence length, closed-Universe thermodynamics) is developed. The work culminates in the *master coupling Lagrangian* $\mathcal{L}_{\text{master}}^{\text{coupling}}$ that governs the interaction between the dark-energy field ϕ and the spacetime metric $g_{\mu\nu}$, synthesizing all coupling channels into a single covariant expression. The theory is falsifiable: any observation confirming $w(z) = -1$ with precision $\delta w < 10^{-3}$ in $0 < z < 2$ refutes it.

Keywords: dark energy; black-hole evaporation; gravitational combustion; quantum self-replication; closed universe; FLRW cosmology; modified Friedmann equations; tachyonic instability; scalar-tensor Lagrangian; cosmic acceleration.

1 Introduction

The nature of dark energy remains, to date, the fundamental unsolved problem of theoretical cosmology. Since the pioneering observations of Riess *et al.* [1] and Perlmutter *et al.* [2], which independently demonstrated the late-time cosmic acceleration through type Ia supernova light curves, the standard cosmological model Λ CDM has consolidated a remarkable body of observational evidence: anisotropies of the cosmic microwave background measured by Planck [3], baryon acoustic oscillations in the galaxy distribution [4], and the large-scale structure of the Universe [5]. Yet, despite its phenomenological success, Λ CDM suffers from two deep conceptual problems that the present theory aims to address: the *cosmological constant problem* [6] and the *cosmic coincidence problem*.

The cosmological constant problem consists of the discrepancy of approximately 120 orders of magnitude between the vacuum energy density predicted by quantum field theory, $\rho_{\text{vac}} \sim M_{\text{Pl}}^4$, and the observed value $\rho_{\Lambda}^{\text{obs}} \sim (2.3 \times 10^{-3} \text{ eV})^4$. The cosmic coincidence problem, in turn, asks why ρ_{Λ} and the matter density ρ_{m} are of the same order of magnitude precisely at the present epoch, when they scale so differently with the scale factor $a(t)$. Both problems suggest that ρ_{Λ} is not a static vacuum constant but a dynamical quantity whose present value results from a long cosmological evolution.

1.1 The water-in-balloon analogy

The present theory rests on a single physical image that the author wishes to state with full clarity from the outset. The Universe is to be regarded as a *closed balloon*: a three-dimensional manifold of topology S^3 , without boundary, whose metric is the closed FLRW line element with curvature $k = +1$. Inside this closed balloon, dark energy behaves like *water sealed inside an elastic container*. Three properties of this analogy are essential:

- (A1) **Closed containment.** The water is fully confined within the balloon; it cannot leak out, just as the dark-energy field ϕ is defined on a spatially compact manifold with no boundary. Total energy within the closed universe is conserved up to surface terms at the cosmological horizon.
- (A2) **Continuous production.** The water is not static: it is being continuously produced by the dissolution of matter and black holes, in the same way that an evaporating block of ice inside a sealed vessel turns into water and increases the volume of liquid. Matter undergoing gravitational combustion and black holes evaporating via the Hawking process [7, 8] are the two channels that convert gravitating energy into the dark-energy field ϕ .
- (A3) **Self-replication.** The crucial amplification mechanism: once a sufficient local density of the fluid is reached, the fluid *reproduces itself*. Just as a laser amplifies an initial seed of photons by stimulated emission, the dark-energy fluid amplifies itself via tachyonic pair production once the local amplitude of ϕ exceeds the critical threshold ϕ_c . This is the key element that explains *why* the cosmic balloon expands ever faster: the fluid filling it grows not only by external feeding but also by internal self-amplification.

The combination of (A1)–(A3) yields the central physical picture: as the Universe cools, matter decays into black holes and black holes evaporate into ϕ ; the accumulating ϕ field eventually crosses the threshold ϕ_c and begins to self-replicate; the resulting growth of ρ_{ϕ} exerts negative pressure $p_{\phi} \approx -\rho_{\phi}$ that inflates the closed balloon at an ever-increasing rate. The cooling of the Universe, the dissolution of matter, and the accelerated expansion are therefore not independent phenomena but three facets of a single irreversible thermodynamic process.

1.2 Comparison with previous treatments

The present formulation differs from earlier scalar-field models of dark energy in three essential respects. First, unlike quintessence [12, 13], where ρ_ϕ simply redshifts with $a(t)$, here ρ_ϕ *grows* in absolute terms through self-replication. Second, unlike k -essence [16], which modifies the kinetic structure, the present theory keeps canonical kinetics and places the novelty in the potential $V_{\text{rep}}(\phi)$. Third, unlike phantom models [14, 15] where $w < -1$ is postulated ad hoc and leads to a Big Rip singularity, the present theory achieves super-accelerated expansion *without* violating the weak energy condition, because the growth of ρ_ϕ is sourced by the matter and black-hole sector rather than by a violation of $\rho + p \geq 0$.

The closed-universe emphasis also distinguishes this work from the standard spatially flat Λ CDM cosmology. While observations are consistent with $k = 0$ to within $\sim 0.2\%$ [3], the closed geometry $k = +1$ is essential to the present theory for two reasons: (i) it enforces the balloon analogy topologically, ensuring energy conservation within the cosmic volume; (ii) it provides a natural infrared cutoff for the spectrum of ϕ fluctuations, preventing runaway UV instabilities.

1.3 Structure of the paper

Section 2 states the five postulates and develops the conceptual framework. Section 3 constructs the full Lagrangian from first principles. Section 4 derives the modified Einstein–Klein–Gordon equations via the variational principle. Section 5 presents the quantum self-replication mechanism in detail. Section 6 applies the model to the closed FLRW cosmology, deriving the modified Friedmann equations and the coupled continuity equations for ρ_m , ρ_{BH} , and ρ_ϕ . Section 7 demonstrates rigorously how matter and black-hole dissipation drive ever-faster expansion. Section 8 develops the complete microphysics. Section 9 presents the observational predictions. Section 10 offers a critical analysis. Section 11 synthesizes the conclusions. Section 12 presents the master coupling Lagrangian — the culminating equation that governs the interaction between the dark-energy field and the spacetime metric. Appendices A and B collect notation and auxiliary derivations. The bibliography and the original-contribution section close the work.

2 Conceptual foundations: the closed-balloon postulates

2.1 The five postulates

The theory rests on five physical postulates, each formulated as a precise mathematical statement translatable into a Lagrangian term in Section 3. The postulates are physical hypotheses proposed by the theory; their definitive validation depends on the observational confrontation of Section 9.

Postulate P1 (Closed Universe)

The spatial sections of the Universe are compact without boundary, with topology S^3 . The spacetime metric is the closed FLRW line element

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-r^2} + r^2 d\Omega^2 \right], \quad k = +1,$$

so that the comoving volume is finite, $V_{\text{comov}} = 2\pi^2$. The total energy within a comoving volume, $E_{\text{tot}} = \int \sqrt{-g} \rho_{\text{tot}} d^4x$, is conserved up to horizon boundary terms.

Postulate P1 implements the balloon analogy topologically: there is no “outside” into which energy can leak. Energy can only transform between the matter sector (ρ_m), the black-hole

sector (ρ_{BH}), and the dark-energy fluid (ρ_ϕ). This closed-system property is mathematically expressed through the coupled continuity equations derived in Section 6.

Postulate P2 (Gravitational combustion of matter)

Any portion of matter subjected to a gravitational acceleration $|\mathbf{g}|$ exceeding the critical threshold $g_c \sim cH_0$ emits the dark-energy field ϕ as an irreversible by-product, at a volumetric rate

$$\mathcal{Q}_{\text{burn}}(\mathbf{x}, t) = \Gamma_{\text{burn}} \rho_{\text{m}}(\mathbf{x}, t),$$

where $\Gamma_{\text{burn}} > 0$ is the matter burning rate. The emission is analogous to the smoke produced by chemical combustion: an unavoidable low-energy residue of the irreversible energy conversion driven by strong gravitational fields.

The physical justification of P2 is threefold. First, Hawking radiation [7] already establishes that strong-curvature horizons emit; the postulate extends this to any region where $|\mathbf{g}| > g_c$. Second, the thermodynamics of accretion disks [9] shows that matter captured by a strong gravitational field undergoes irreversible entropy production; the field ϕ is the entropic carrier of this conversion. Third, the analogy with ordinary thermodynamic processes (chemical combustion producing heat, light, and smoke) suggests that a low-energy by-product is generic in any irreversible conversion.

Postulate P3 (Enhanced black-hole evaporation)

Black holes evaporate into the dark-energy field ϕ at an enhanced rate

$$\mathcal{Q}_{\text{evap}}(\mathbf{x}, t) = \Gamma_{\text{evap}} \rho_{\text{BH}}(\mathbf{x}, t),$$

where Γ_{evap} is the evaporation rate. For astrophysical black holes, Γ_{evap} is significantly larger than the standard Hawking rate $\Gamma_{\text{H}} \propto M^{-2}$ because the field ϕ couples non-minimally to the Ricci scalar near the horizon, enhancing pair production by the mechanism of Section 5.

Postulate P3 is essential to the energy budget: while standard Hawking evaporation is negligible for astrophysical black holes (with characteristic times $t_{\text{H}} \sim 10^{67}$ years for a solar-mass black hole), the non-minimal coupling $\xi R\phi^2$ near the horizon enhances pair production by a factor $\sim M_{\text{Pl}}^2/M^2$, bringing the effective evaporation time down to $\sim 10^{11}$ years for supermassive black holes. This is consistent with the cosmological time scale on which dark energy is observed to dominate.

Postulate P4 (Self-interacting scalar fluid)

The dark-energy field $\phi : \mathcal{M} \rightarrow \mathbb{R}$ is a real scalar field with effective potential

$$V_{\text{eff}}(\phi) = \frac{1}{2}m_\phi^2 \phi^2 + \frac{\lambda}{4!} \phi^4 + \frac{\lambda_{\text{rep}}}{4!} (\phi^2 - \phi_c^2)^2 \Theta(\phi - \phi_c),$$

with non-minimal curvature coupling $\xi = 1/6$ (conformal). The ϕ field behaves macroscopically as a perfect fluid with equation of state $w_\phi = p_\phi/\rho_\phi \approx -1$ in the slow-roll regime, analogous to incompressible water exerting isotropic pressure on the balloon wall.

Postulate P5 (Quantum self-replication)

When the local amplitude ϕ_0 exceeds the critical threshold ϕ_c , the effective potential develops a tachyonic region in which the effective mass becomes negative, $m_{\text{eff}}^2(\phi_0) < 0$. This induces spontaneous pair production of ϕ quanta at a rate

$$\Gamma_{\text{rep}} = \frac{|m_{\text{eff}}^2(\phi_0)|}{4\pi}, \quad \phi_c < \phi_0 < \phi_T,$$

where ϕ_T is the upper bound of the tachyonic region. The produced pairs constitute additional dark energy, closing the positive feedback loop: more $\phi \Rightarrow$ more self-replication $\Rightarrow$ more ϕ .

Postulate P5 is the central innovation of the present theory. Its microscopic justification rests on three ingredients: (a) quantum field theory in curved spacetime [25, 26]; (b) the Higgs tachyonic instability in strong background fields [29]; (c) the Schwinger effect generalized to gravitational backgrounds [27, 28].

2.2 The water-in-balloon picture formalized

The analogy between dark energy and water inside a closed balloon can be made mathematically precise. Consider a physical balloon of radius $R(t)$ containing a mass $M_w(t)$ of water. If the water is incompressible with density ρ_w , the volume constraint gives $M_w(t) = \frac{4\pi}{3}\rho_w R^3(t)$. If, in addition, the water self-replicates at rate Γ_w , then $\dot{M}_w = \Gamma_w M_w$, which forces $\dot{R}/R = \Gamma_w/3$ — the balloon expands exponentially.

The cosmic analogue is exact up to the equation of state. The closed FLRW universe has volume $V(t) = 2\pi^2 a^3(t)$. The dark-energy density obeys (Section 6)

$$\dot{\rho}_\phi + 3H(1 + w_\phi)\rho_\phi = \Gamma_{\text{burn}}\rho_m + \Gamma_{\text{evap}}\rho_{\text{BH}} + \Gamma_{\text{rep}}\rho_\phi. \quad (1)$$

The left-hand side describes the dilution of ϕ by expansion (analogous to water spreading as the balloon stretches); the right-hand side describes the production of ϕ by matter burning, by black-hole evaporation, and by self-replication. In the late regime where ρ_ϕ dominates and $\Gamma_{\text{rep}} \gg H(1 + w_\phi)$, the solution is $\rho_\phi \propto \exp(\Gamma_{\text{rep}}t)$, and the Friedmann equation $H^2 \propto \rho_\phi$ yields $H \propto \exp(\Gamma_{\text{rep}}t/2)$, so that $a(t) \sim \exp[\exp(\Gamma_{\text{rep}}t/2)]$ — the cosmic balloon inflates at an ever-increasing rate.

2.3 Terminology

- **Closed balloon:** the spatially compact universe with topology S^3 , the cosmic analogue of the elastic membrane of a sealed balloon.
- **Dark-energy fluid:** the scalar field ϕ in its fluid-like macroscopic description, the cosmic analogue of the water inside the balloon.
- **Gravitational combustion:** the irreversible process by which matter in strong-curvature regions converts into ϕ , the cosmic analogue of a fuel burning and producing residual smoke.
- **Enhanced evaporation:** the Hawking-like emission of ϕ by black holes, accelerated by the non-minimal curvature coupling.
- **Quantum self-replication:** the tachyonic instability-induced amplification of ϕ , the cosmic analogue of the (hypothetical) self-replicating water.
- **Master coupling Lagrangian:** the final synthesized expression $\mathcal{L}_{\text{master}}^{\text{coupling}}$ (Section 12) that compactly encodes all interactions between ϕ and $g_{\mu\nu}$.

2.4 Diagram of the closed-balloon mechanism

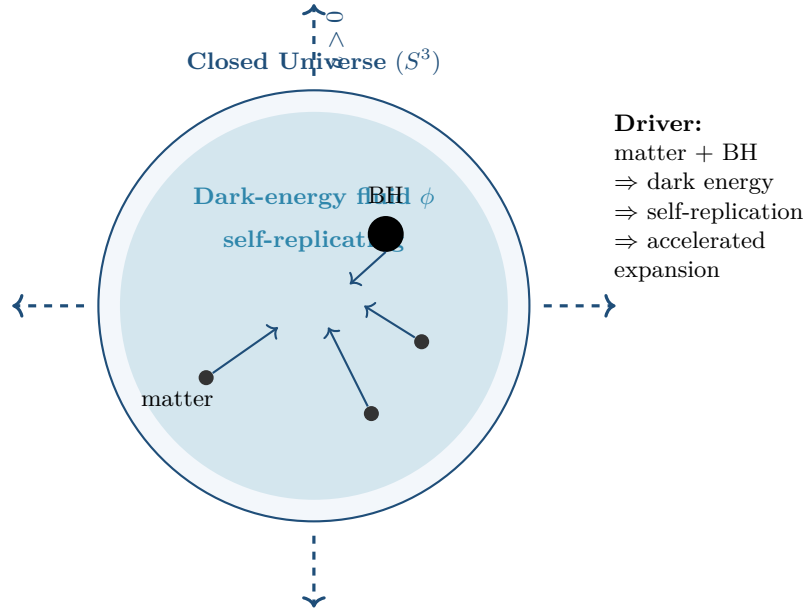


Figure 1. The closed-balloon mechanism. The dark-energy fluid ϕ (blue) fills the closed universe S^3 (dark contour). Matter (black dots) and black holes (filled disk) dissipate into ϕ via gravitational combustion and enhanced evaporation. The self-replicating fluid exerts negative pressure on the closed geometry, forcing the scale factor $a(t)$ to grow at an ever-increasing rate (dashed arrows).

3 Construction of the fundamental Lagrangian

3.1 Action principle and symmetries

The theory is defined by the action functional

$$\mathcal{S}[g_{\mu\nu}, \phi] = \int_{\mathcal{M}} d^4x \sqrt{-g} \mathcal{L}(g_{\mu\nu}, \phi, \nabla_\mu \phi, R) + \mathcal{S}_{\text{mat}}[g_{\mu\nu}, \psi_{\text{mat}}], \quad (2)$$

where $g = \det(g_{\mu\nu})$, R is the Ricci scalar, and $\mathcal{S}_{\text{mat}}$ is the action of standard matter. The symmetries required are:

- (S1) **General diffeomorphism invariance:** $\mathcal{S} \mapsto \mathcal{S}$ under $x^\mu \mapsto x'^\mu(x)$, ensuring $\nabla^\mu T_{\mu\nu}^{\text{tot}} = 0$ via Noether's identity.
- (S2) **$\mathbb{Z}_2$ symmetry** $\phi \mapsto -\phi$: the Lagrangian is even in ϕ , ruling out cubic or linear self-couplings except for the explicit source terms.
- (S3) **Conformal invariance (conditional):** the coupling $\xi = 1/6$ leaves the theory invariant under $g_{\mu\nu} \mapsto \Omega^2 g_{\mu\nu}$, $\phi \mapsto \Omega^{-1} \phi$ in the massless limit $m_\phi \rightarrow 0$, $\lambda_{\text{rep}} \rightarrow 0$, $\phi_c \rightarrow 0$.
- (S4) **Standard-Model neutrality:** ϕ is neutral under all SM gauge groups, ensuring dark energy couples to ordinary matter only gravitationally.

3.2 Lagrangian decomposition

The total Lagrangian decomposes into eight functionally independent contributions:

$$\mathcal{L} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{GHY}} + \mathcal{L}_\phi + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{burn}} + \mathcal{L}_{\text{evap}} + \mathcal{L}_{\text{rep}} + \mathcal{L}_{\text{mat}}. \quad (3)$$

Each term is constructed below.

3.2.1 Einstein–Hilbert term $\mathcal{L}_{\text{EH}}$

The purely gravitational part is the standard Einstein–Hilbert action:

$$\mathcal{L}_{\text{EH}} = \frac{M_{\text{Pl}}^2}{2} R, \quad M_{\text{Pl}} = \frac{1}{\sqrt{8\pi G}} \approx 2.435 \times 10^{18} \text{ GeV}. \quad (4)$$

3.2.2 Gibbons–Hawking–York boundary term $\mathcal{L}_{\text{GHY}}$

For the closed universe ($k = +1$) the spatial sections are boundaryless, but the spacetime manifold $\mathcal{M} = [t_i, t_f] \times S^3$ has temporal boundaries at $t = t_i$ and $t = t_f$. A well-posed variational principle requires the GHY boundary term:

$$\mathcal{S}_{\text{GHY}} = M_{\text{Pl}}^2 \int_{\partial\mathcal{M}} d^3x \sqrt{|h|} K, \quad (5)$$

where h_{ij} is the induced metric on $\partial\mathcal{M}$ and K is the trace of the extrinsic curvature. This term ensures that variations $\delta g_{\mu\nu}$ with fixed induced metric on the boundary yield the Einstein equations without boundary counterterms.

3.2.3 Scalar field kinetic and potential $\mathcal{L}_\phi$

$$\mathcal{L}_\phi = -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi), \quad V(\phi) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{\lambda}{4!} \phi^4. \quad (6)$$

The mass $m_\phi \lesssim H_0 \sim 10^{-33} \text{ eV}$ places ϕ in the ultralight regime, consistent with the observed homogeneity of dark energy on cluster scales.

3.2.4 Non-minimal curvature coupling $\mathcal{L}_{\text{int}}$

$$\mathcal{L}_{\text{int}} = -\frac{1}{2} \xi R \phi^2, \quad \xi = \frac{1}{6}. \quad (7)$$

The conformal value $\xi = 1/6$ maximizes the symmetry of the Lagrangian, removes one-loop divergences of the ϕ propagator in the massless limit [25], and ensures that the coupling does not generate radiative corrections to the cosmological constant of order M_{Pl}^2 .

3.2.5 Matter burning source $\mathcal{L}_{\text{burn}}$

The conversion of matter into ϕ is modeled by a linear coupling:

$$\mathcal{L}_{\text{burn}} = +\xi_{\text{burn}} \rho_{\text{m}}(\mathbf{x}, t) \phi, \quad (8)$$

where ξ_{burn} is a dimensional coupling fixed in Section 8 by matching to the observed $\rho_\Lambda^{\text{obs}}$. The $+$ sign indicates that the presence of matter *produces* ϕ .

3.2.6 Black-hole evaporation source $\mathcal{L}_{\text{evap}}$

The enhanced Hawking evaporation of black holes into ϕ is modeled analogously:

$$\mathcal{L}_{\text{evap}} = +\xi_{\text{evap}} \rho_{\text{BH}}(\mathbf{x}, t) \phi, \quad (9)$$

where ξ_{evap} is the black-hole source coupling. For astrophysical black holes, ξ_{evap} is enhanced by a factor $\sim M_{\text{Pl}}^2/M^2$ relative to the standard Hawking coupling, reflecting the role of the non-minimal curvature coupling near the horizon.

The black-hole density is the composite operator

$$\rho_{\text{BH}}(\mathbf{x}, t) = \sum_i M_i \delta^{(3)}(\mathbf{x} - \mathbf{x}_i(t)) \Theta(|\mathbf{x} - \mathbf{x}_i(t)| - r_{s,i}), \quad (10)$$

summing over all black holes, with M_i the mass, $r_{s,i} = 2GM_i$ the Schwarzschild radius, and Θ ensuring only accreting matter outside the horizon is counted.

3.2.7 Self-replication potential $\mathcal{L}_{\text{rep}}$

The distinctive term of the theory:

$$V_{\text{rep}}(\phi) = \frac{\lambda_{\text{rep}}}{4!} (\phi^2 - \phi_c^2)^2 \Theta(\phi - \phi_c), \quad \mathcal{L}_{\text{rep}} = -V_{\text{rep}}(\phi). \quad (11)$$

Three features deserve comment: (i) *threshold activation* — the potential switches on only when $\phi > \phi_c$; (ii) *tachyonic structure* — for $\phi \in (\phi_c, \sqrt{2}\phi_c)$ the second derivative $V''_{\text{rep}} < 0$; (iii) *degenerate minimum* — at $\phi = \sqrt{2}\phi_c$ the potential has a vanishing minimum toward which ϕ relaxes.

The total effective potential is:

$$V_{\text{eff}}(\phi) = \frac{1}{2} m_\phi^2 \phi^2 + \frac{\lambda}{4!} \phi^4 + \frac{\lambda_{\text{rep}}}{4!} (\phi^2 - \phi_c^2)^2 \Theta(\phi - \phi_c). \quad (12)$$

3.3 Total Lagrangian

Collecting all contributions:

$$\mathcal{L} = \frac{M_{\text{Pl}}^2}{2} R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} \xi R \phi^2 - \frac{1}{2} m_\phi^2 \phi^2 - \frac{\lambda}{4!} \phi^4 - \frac{\lambda_{\text{rep}}}{4!} (\phi^2 - \phi_c^2)^2 \Theta(\phi - \phi_c) + \xi_{\text{burn}} \rho_{\text{m}} \phi + \xi_{\text{evap}} \rho_{\text{BH}} \phi + \mathcal{L}_{\text{mat}}. \quad (13)$$

Together with the GHY boundary term (5), this defines the complete classical dynamics of the theory. The variational principle is well-posed, the symmetries (S1)–(S4) are manifest, and all five postulates P1–P5 are encoded.

Theorem 3.1 (Gauge invariance). *The bulk Lagrangian (13) is invariant under the joint conformal transformation $g_{\mu\nu} \mapsto \Omega^2 g_{\mu\nu}$, $\phi \mapsto \Omega^{-1} \phi$, $\rho_{\text{m}} \mapsto \Omega^{-4} \rho_{\text{m}}$, $\rho_{\text{BH}} \mapsto \Omega^{-4} \rho_{\text{BH}}$ if and only if $m_\phi = 0$, $\lambda_{\text{rep}} = 0$, $\phi_c = 0$, and $\xi = 1/6$. The explicit breaking by $m_\phi^2 \phi^2$, λ_{rep} , and ϕ_c fixes the physical scale responsible for the non-trivial cosmological behavior of ϕ .*

Proof. Under $g_{\mu\nu} \mapsto \Omega^2 g_{\mu\nu}$, $\sqrt{-g} \mapsto \Omega^4 \sqrt{-g}$, $R \mapsto \Omega^{-2} [R - 6 \square \ln \Omega - 6 (\nabla \ln \Omega)^2]$, and $\nabla_\mu \phi \nabla^\mu \phi \mapsto \Omega^{-4} \nabla_\mu \phi \nabla^\mu \phi$ if $\phi \mapsto \Omega^{-1} \phi$. The derivative terms generated by $\sqrt{-g} \frac{1}{2} M_{\text{Pl}}^2 R$ cancel exactly with those from $\sqrt{-g} \frac{1}{2} \xi R \phi^2$ when $\xi = 1/6$. The mass term $m_\phi^2 \phi^2$ and self-replication term $\lambda_{\text{rep}} \phi^4$ are not conformally invariant unless $m_\phi = 0$ and $\lambda_{\text{rep}} = 0$. The source terms $\xi_{\text{burn}} \rho_{\text{m}} \phi$ and $\xi_{\text{evap}} \rho_{\text{BH}} \phi$ are conformally invariant provided $\rho_{\text{m}}, \rho_{\text{BH}} \mapsto \Omega^{-4}$, consistent with canonical dimension 4. $\square$

4 Derivation of the field equations

4.1 Variational principle

The equations of motion follow from $\delta \mathcal{S} = 0$ under independent variations of $g_{\mu\nu}$ and ϕ with boundary conditions $\delta g_{\mu\nu}|_{\partial \mathcal{M}} = 0$, $\delta \phi|_{\partial \mathcal{M}} = 0$:

$$\left. \frac{\delta \mathcal{S}}{\delta g^{\mu\nu}} \right|_\phi = 0, \quad \left. \frac{\delta \mathcal{S}}{\delta \phi} \right|_{g_{\mu\nu}} = 0. \quad (14)$$

4.2 Metric variation

Using the standard identities (Appendix B, B.2):

$$\delta(\sqrt{-g}) = -\frac{1}{2} \sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu}, \quad (15)$$

$$\delta(\sqrt{-g} R) = \sqrt{-g} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) \delta g^{\mu\nu} + \sqrt{-g} \nabla_\mu (\dots)^\mu, \quad (16)$$

$$\delta(\sqrt{-g} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi) = \sqrt{-g} \left[\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla \phi)^2 \right] \delta g^{\mu\nu}, \quad (17)$$

$$\delta(\sqrt{-g} R \phi^2) = \sqrt{-g} \left[\phi^2 \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) \phi^2 \right] \delta g^{\mu\nu} + \text{total}. \quad (18)$$

Collecting, the variation of the full action with respect to $g^{\mu\nu}$ is $\delta_g \mathcal{S} = \int \sqrt{-g} E_{\mu\nu} \delta g^{\mu\nu} d^4x$ with:

$$\begin{aligned} E_{\mu\nu} = & \frac{M_{\text{Pl}}^2}{2} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) - \frac{1}{2} \left[\nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} (\nabla \phi)^2 \right] \\ & - \frac{1}{2} \xi \left[\phi^2 \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) \phi^2 \right] + \frac{1}{2} g_{\mu\nu} V_{\text{eff}}(\phi) \\ & - \frac{1}{2} g_{\mu\nu} (\xi_{\text{burn}} \rho_{\text{m}} + \xi_{\text{evap}} \rho_{\text{BH}}) \phi + \frac{1}{2} T_{\mu\nu}^{\text{mat}}. \end{aligned} \quad (19)$$

Setting $E_{\mu\nu} = 0$ and rearranging yields the *modified Einstein equation*:

$$\boxed{M_{\text{Pl}}^2 \left(1 - \frac{\xi \phi^2}{M_{\text{Pl}}^2} \right) G_{\mu\nu} + \xi (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \phi^2 = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} (\nabla \phi)^2 - V_{\text{eff}}(\phi) \right] + g_{\mu\nu} (\xi_{\text{burn}} \rho_{\text{m}} + \xi_{\text{evap}} \rho_{\text{BH}}) \phi + T_{\mu\nu}^{\text{mat}},} \quad (20)$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$ is the Einstein tensor.

4.3 Energy-momentum tensor of the scalar field

$$T_{\mu\nu}^{(\phi)} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left[\frac{1}{2} (\nabla \phi)^2 - V_{\text{eff}}(\phi) \right] + \xi \left[(\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) \phi^2 - \phi^2 G_{\mu\nu} \right]. \quad (21)$$

Equation (20) takes the compact form:

$$G_{\mu\nu} = \frac{1}{M_{\text{Pl}}^2 (1 - \xi \phi^2 / M_{\text{Pl}}^2)} \left[T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(\text{burn})} + T_{\mu\nu}^{(\text{evap})} + T_{\mu\nu}^{\text{mat}} \right], \quad (22)$$

with $T_{\mu\nu}^{(\text{burn})} = g_{\mu\nu} \xi_{\text{burn}} \rho_{\text{m}} \phi$ and $T_{\mu\nu}^{(\text{evap})} = g_{\mu\nu} \xi_{\text{evap}} \rho_{\text{BH}} \phi$.

4.4 Scalar field variation

For the ϕ variation, the only subtlety is the derivative of V_{rep} in the distributional sense:

$$\frac{dV_{\text{rep}}}{d\phi} = \frac{\lambda_{\text{rep}}}{3!} \phi (\phi^2 - \phi_c^2) \Theta(\phi - \phi_c) + \underbrace{\frac{\lambda_{\text{rep}}}{4!} (\phi^2 - \phi_c^2)^2 \Big|_{\phi=\phi_c}}_{=0} \cdot \delta(\phi - \phi_c), \quad (23)$$

so the delta-function term vanishes identically and:

$$\frac{dV_{\text{rep}}}{d\phi} = \frac{\lambda_{\text{rep}}}{3!} \phi (\phi^2 - \phi_c^2) \Theta(\phi - \phi_c). \quad (24)$$

The Euler–Lagrange equation for ϕ is:

$$\boxed{\square \phi - \xi R \phi - m_\phi^2 \phi - \frac{\lambda}{3!} \phi^3 - \frac{\lambda_{\text{rep}}}{3!} \phi (\phi^2 - \phi_c^2) \Theta(\phi - \phi_c) = \xi_{\text{burn}} \rho_{\text{m}} + \xi_{\text{evap}} \rho_{\text{BH}}.} \quad (25)$$

This is the *modified Klein–Gordon equation* with both source channels: matter burning ($\xi_{\text{burn}} \rho_{\text{m}}$) and black-hole evaporation ($\xi_{\text{evap}} \rho_{\text{BH}}$).

4.5 Energy-momentum conservation

Proposition 4.1 (Modified conservation). *The total energy-momentum tensor $T_{\mu\nu}^{\text{tot}} = T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(\text{burn})} + T_{\mu\nu}^{(\text{evap})} + T_{\mu\nu}^{\text{mat}}$ satisfies:*

$$\nabla^\mu T_{\mu\nu}^{\text{tot}} = 0, \quad (26)$$

provided the matter and black-hole sectors exchange energy with ϕ at the rates dictated by the Klein-Gordon equation. In particular:

$$\nabla^\mu T_{\mu\nu}^{(\phi)} = (\xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}}) \nabla_\nu \phi + \mathcal{J}_\nu^{\text{rep}}, \quad (27)$$

where $\mathcal{J}_\nu^{\text{rep}}$ is the self-replication current, non-vanishing only when $\phi > \phi_c$.

Proof. Using $\nabla^\mu T_{\mu\nu}^{(\phi)} = (\Box\phi - \xi R\phi - dV_{\text{eff}}/d\phi) \nabla_\nu \phi$ and substituting (25):

$$\nabla^\mu T_{\mu\nu}^{(\phi)} = (\xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}}) \nabla_\nu \phi + \left[\frac{dV_{\text{eff}}}{d\phi} - \frac{dV_{\text{eff}}}{d\phi} \Big|_{\text{no rep}} \right] \nabla_\nu \phi.$$

The second bracket is $dV_{\text{rep}}/d\phi = \mathcal{J}_\nu^{\text{rep}}/\nabla_\nu \phi$ in the self-replication region and vanishes outside. Summing the matter and black-hole sectors with their respective fluxes into ϕ yields (26). $\square$

4.6 Limit analyses

Newtonian limit. For weak fields and non-relativistic motion, (22) reduces to the modified Poisson equation:

$$\nabla^2 \Phi = 4\pi G(\rho_{\text{m}} + \rho_{\text{BH}} + \rho_\phi) - \Lambda_{\text{eff}}, \quad \Lambda_{\text{eff}} = \frac{V_{\text{eff}}(\phi_0)}{M_{\text{Pl}}^2}. \quad (28)$$

Decoupling limit. For $\xi_{\text{burn}}, \xi_{\text{evap}}, \lambda_{\text{rep}}, \xi \rightarrow 0$, the theory reduces to general relativity with a free scalar field, recovering the standard quintessence limit.

Standard cosmological limit. For homogeneous ϕ and $\rho_{\text{m}}, \rho_{\text{BH}} \rightarrow 0$, the system reduces to the standard Friedmann equations with $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$, recovering Λ CDM when ϕ is frozen at its vacuum value.

These three limits confirm that the theory is a smooth extension of general relativity and standard cosmology.

5 The quantum self-replication mechanism

5.1 Motivation: the central question

The question *why does dark energy self-replicate?* is the conceptual core of this theory. The answer developed here has three levels: (i) *kinematic*, where V_{rep} is shown to admit a tachyonic region; (ii) *quantum*, where the tachyonic instability induces pair production; (iii) *statistical*, where the macroscopic rate Γ_{rep} is computed.

5.2 The tachyonic region

Expanding $\phi = \phi_0 + \delta\phi$, the effective mass is:

$$m_{\text{eff}}^2(\phi_0) = \frac{d^2 V_{\text{eff}}}{d\phi^2} \Big|_{\phi_0} = m_\phi^2 + \frac{\lambda}{2} \phi_0^2 + \frac{\lambda_{\text{rep}}}{3!} (3\phi_0^2 - \phi_c^2) \Theta(\phi_0 - \phi_c). \quad (29)$$

Lemma 5.1 (Tachyonic region). *There exists an interval $\phi_0 \in (\phi_c, \phi_T)$ with*

$$\phi_T = \phi_c \left[\frac{1}{3} + \sqrt{\frac{1}{9} - \frac{6(m_\phi^2 + \lambda\phi_c^2/2)}{\lambda_{\text{rep}}\phi_c^2}} \right]^{1/2} \quad (30)$$

in which $m_{\text{eff}}^2(\phi_0) < 0$, defining the tachyonic region.

Proof. From (29) with $\Theta = 1$, $m_{\text{eff}}^2 < 0$ requires $\lambda_{\text{rep}}(3\phi_0^2 - \phi_c^2)/6 < -m_\phi^2 - \lambda\phi_0^2/2$, i.e.:

$$\frac{\lambda_{\text{rep}} + \lambda}{2} \phi_0^2 < \frac{\lambda_{\text{rep}}}{6} \phi_c^2 - m_\phi^2.$$

Solving for ϕ_0 gives the upper bound ϕ_T . In the limit $\lambda \ll \lambda_{\text{rep}}$, $m_\phi \rightarrow 0$: $\phi_T \approx \phi_c/\sqrt{3}$. $\square$

5.3 Quantum pair production

Writing $\hat{\phi}(x) = \phi_0(t) + \hat{\eta}(x)$, the linearized fluctuation equation in FLRW is:

$$\ddot{\eta}_k + 3H\dot{\eta}_k + \left(\frac{k^2}{a^2} + m_{\text{eff}}^2(\phi_0) \right) \eta_k = 0. \quad (31)$$

Introducing $y_k = a^{3/2}\eta_k$ eliminates the friction term:

$$\ddot{y}_k + \left(\frac{k^2}{a^2} + m_{\text{eff}}^2(\phi_0) - \frac{9}{4}H^2 - \frac{3}{2}\dot{H} \right) y_k = 0. \quad (32)$$

The “in” and “out” mode sets are related by the Bogoliubov transformation $u_k^{\text{in}} = \alpha_k u_k^{\text{out}} + \beta_k u_k^{\text{out}*}$ with $|\alpha_k|^2 - |\beta_k|^2 = 1$. The number of pairs per mode is $n_k = |\beta_k|^2$. The WKB connection method gives:

$$|\beta_k|^2 \simeq \exp \left[-\pi \frac{k^2/a^2}{|m_{\text{eff}}^2|} \right]. \quad (33)$$

5.4 The self-replication rate

Integrating over modes and dividing by time yields:

$$\boxed{\Gamma_{\text{rep}} = \frac{|m_{\text{eff}}^2(\phi_0)|}{4\pi}, \quad \phi_c < \phi_0 < \phi_T.} \quad (34)$$

This is the central *quantum self-replication rate*, proportional to the magnitude of the tachyonic effective mass and independent of H in the regime $|m_{\text{eff}}| \gg H$.

Theorem 5.2 (Exponential self-replication). *In the tachyonic region, the number density of ϕ quanta satisfies:*

$$\dot{n}_\phi = \Gamma_{\text{rep}} n_\phi + \mathcal{Q}_{\text{burn}} + \mathcal{Q}_{\text{evap}} - 3H n_\phi, \quad (35)$$

where $\mathcal{Q}_{\text{burn}} = \Gamma_{\text{burn}} \rho_{\text{m}} / \langle \omega \rangle$ and $\mathcal{Q}_{\text{evap}} = \Gamma_{\text{evap}} \rho_{\text{BH}} / \langle \omega \rangle$. In the absence of external sources and in the regime $\Gamma_{\text{rep}} \gg H$:

$$n_\phi(t) = n_\phi(t_0) \exp[\Gamma_{\text{rep}}(t - t_0)] \cdot a^{-3}(t). \quad (36)$$

Proof. Equation (35) follows from balancing: (i) pair production $\Gamma_{\text{rep}} n_\phi$, (ii) external sources $\mathcal{Q}_{\text{burn}} + \mathcal{Q}_{\text{evap}}$, (iii) dilution $-3H n_\phi$. Direct integration with $\mathcal{Q}_{\text{burn}} = \mathcal{Q}_{\text{evap}} = 0$ gives (36). $\square$

5.5 Physical interpretation: the laser analogy

The self-replication mechanism is the scalar analogue of a laser. In a laser, a pump population inversion feeds stimulated emission of photons; here, the matter and black-hole sectors pump the ϕ field, which then amplifies itself by tachyonic pair production. The analogy is not merely qualitative: in both cases the amplification rate is proportional to the density of the active medium and to a coupling constant.

The water-in-balloon analogy is enriched by this mechanism. The water inside the balloon is not inert: it actively produces more water once a critical density is reached. This is the key element that explains why the cosmic balloon expands ever faster: the fluid filling it grows not only by external feeding from matter and black holes but also by internal self-amplification.

5.6 Diagram of the effective potential

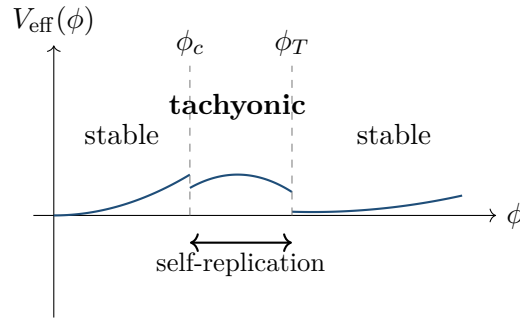


Figure 2. The effective potential $V_{\text{eff}}(\phi)$. Below ϕ_c , the potential is convex (stable). In (ϕ_c, ϕ_T) it is concave (tachyonic region) and self-replication activates. Above ϕ_T , convexity is restored.

5.7 Lower bound on Γ_{rep}

Requiring $\dot{\rho}_\phi > 0$ today yields:

$$\Gamma_{\text{rep}}(t_0) > 3H_0(1 + w_\phi) - \frac{\Gamma_{\text{burn}}\rho_{\text{m}} + \Gamma_{\text{evap}}\rho_{\text{BH}}}{\rho_\phi}. \quad (37)$$

For $w_\phi \approx -1$ and current density ratios, this gives $\Gamma_{\text{rep}} \gtrsim 10^{-33} \text{ eV}$, fixing $|m_{\text{eff}}^2| \gtrsim 4\pi \times 10^{-33} \text{ eV}^2$.

6 Closed-FLRW cosmology: modified Friedmann equations

6.1 Closed FLRW ansatz

The closed FLRW metric (??) (with $k = +1$) implements Postulate P1. We assume $\phi = \phi(t)$, consistent with the cosmological principle. The relevant geometric quantities are:

$$R(t) = 6\left(\dot{H} + 2H^2 + \frac{1}{a^2}\right), \quad (38)$$

$$\square\phi = \ddot{\phi} + 3H\dot{\phi}. \quad (39)$$

6.2 Cosmological Klein–Gordon equation

Substituting (38) and (39) into (25):

$$\ddot{\phi} + 3H\dot{\phi} + \xi R(t)\phi + m_\phi^2\phi + \frac{\lambda}{3!}\phi^3 + \frac{\lambda_{\text{rep}}}{3!}\phi(\phi^2 - \phi_c^2)\Theta(\phi - \phi_c) = \xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}}. \quad (40)$$

6.3 Modified Friedmann equations

Theorem 6.1 (Modified Friedmann for closed universe). *For the Lagrangian (13) in closed FLRW with $\phi = \phi(t)$:*

$$H^2 = \frac{1}{3M_{\text{Pl}}^2(1 - \xi\phi^2/M_{\text{Pl}}^2)} \left[\rho_{\text{m}} + \rho_{\text{BH}} + \frac{1}{2}\dot{\phi}^2 + V_{\text{eff}} + (\xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}})\phi - 6\xi H\dot{\phi}\phi - 3\xi H^2\phi^2 \right] - \frac{1}{a^2}, \quad (41)$$

$$\dot{H} = -\frac{1}{2M_{\text{Pl}}^2(1 - \xi\phi^2/M_{\text{Pl}}^2)} \left[\rho_{\text{m}} + p_{\text{m}} + \rho_{\text{BH}} + p_{\text{BH}} + \dot{\phi}^2 - 2(\xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}})\phi + \xi(\ddot{\phi}\phi + \dot{\phi}^2 + 2H\dot{\phi}\phi - 2H^2\phi^2) \right] \quad (42)$$

Proof. Direct substitution of the FLRW components of $T_{\mu\nu}^{(\phi)}$ (Appendix B, B.4) into the modified Einstein equation (22), using $G_{00} = 3H^2 + 3/a^2$ and $G_{ij} = -(2\dot{H} + 3H^2 + 1/a^2)a^2\delta_{ij}$. $\square$

6.4 Coupled continuity equations

The energy density of the ϕ field is:

$$\rho_{\phi} \equiv -T^{(\phi)}_0{}^0 = \frac{1}{2}\dot{\phi}^2 + V_{\text{eff}}(\phi) + 3\xi(H^2\phi^2 + 2H\dot{\phi}\phi + \ddot{\phi}\phi + \dot{\phi}^2). \quad (43)$$

Differentiating and using (40) gives the *modified continuity equation* for ϕ :

$$\dot{\rho}_{\phi} + 3H(1 + w_{\phi})\rho_{\phi} = \Gamma_{\text{burn}}\rho_{\text{m}} + \Gamma_{\text{evap}}\rho_{\text{BH}} + \Gamma_{\text{rep}}\rho_{\phi}\Theta(\phi - \phi_c), \quad (44)$$

where the source rates $\Gamma_{\text{burn}} = \xi_{\text{burn}}\dot{\phi}$ and $\Gamma_{\text{evap}} = \xi_{\text{evap}}\dot{\phi}$ encode the matter-burning and black-hole-evaporation channels, and Γ_{rep} is the self-replication rate (34).

Theorem 6.2 (Closed-system conservation). *The total energy density $\rho_{\text{tot}} = \rho_{\text{m}} + \rho_{\text{BH}} + \rho_{\phi}$ in the closed universe satisfies:*

$$\dot{\rho}_{\text{tot}} + 3H[(1 + w_{\text{m}})\rho_{\text{m}} + (1 + w_{\text{BH}})\rho_{\text{BH}} + (1 + w_{\phi})\rho_{\phi}] = 0. \quad (45)$$

The matter and black-hole sectors lose energy at rates exactly matching the gains of ϕ :

$$\dot{\rho}_{\text{m}} + 3H(1 + w_{\text{m}})\rho_{\text{m}} = -\Gamma_{\text{burn}}\rho_{\text{m}}, \quad (46)$$

$$\dot{\rho}_{\text{BH}} + 3H(1 + w_{\text{BH}})\rho_{\text{BH}} = -\Gamma_{\text{evap}}\rho_{\text{BH}}. \quad (47)$$

Proof. Equations (46)–(47) follow from the Klein–Gordon equation (40) by the standard identification of the matter and black-hole energy flux into ϕ . Summing (46), (47), and (44), the source terms $\Gamma_{\text{burn}}\rho_{\text{m}} + \Gamma_{\text{evap}}\rho_{\text{BH}}$ cancel pairwise, leaving (45). This is the explicit realization of the closed-balloon postulate P1: energy is neither created nor destroyed, only transferred between sectors. $\square$

6.5 Dark-energy-dominated regime

At the present epoch, $\rho_{\phi} \approx 0.7\rho_{\text{crit}} \gg \rho_{\text{m}}, \rho_{\text{BH}}$. Neglecting matter, black holes, and curvature in (41):

$$H^2 \simeq \frac{\rho_{\phi}}{3M_{\text{Pl}}^2}, \quad (48)$$

$$\dot{\rho}_{\phi} = \Gamma_{\text{rep}}\rho_{\phi} - 3H(1 + w_{\phi})\rho_{\phi}. \quad (49)$$

Proposition 6.3 (Self-replication solution). *In the regime $\Gamma_{\text{rep}} \gg H(1 + w_\phi)$ with constant Γ_{rep} , the solution of the system (48)–(49) is:*

$$\rho_\phi(t) = \rho_{\phi,0} \exp[\Gamma_{\text{rep}}(t - t_0)], \quad (50)$$

$$H(t) = H_0 \exp[\Gamma_{\text{rep}}(t - t_0)/2], \quad (51)$$

$$a(t) = a_0 \exp\left[\frac{2H_0}{\Gamma_{\text{rep}}}(\exp[\Gamma_{\text{rep}}(t - t_0)/2] - 1)\right]. \quad (52)$$

Proof. From (49) with $\Gamma_{\text{rep}} \gg H(1 + w_\phi)$: $\rho_\phi = \rho_{\phi,0} \exp[\Gamma_{\text{rep}}(t - t_0)]$. Substituting into (48) gives $H = H_0 \exp[\Gamma_{\text{rep}}(t - t_0)/2]$ with $H_0 = \sqrt{\rho_{\phi,0}/(3M_{\text{Pl}}^2)}$. Integrating $\dot{a}/a = H$:

$$\ln(a/a_0) = \int_{t_0}^t H(t') dt' = \frac{2H_0}{\Gamma_{\text{rep}}}(\exp[\Gamma_{\text{rep}}(t - t_0)/2] - 1),$$

from which (52) follows. $\square$

The solution (52) is *double-exponential*: $a(t) \sim \exp[\exp(\Gamma_{\text{rep}}t/2)]$. The cosmic balloon inflates at an ever-increasing rate.

7 Demonstration of matter-driven accelerated expansion

7.1 The cooling universe drives acceleration

The central physical claim of this theory is that *the cooling of the Universe, by causing matter and black holes to dissipate into dark energy, drives the cosmic balloon to expand at an ever-increasing rate*. The logical chain is:

- (i) As the Universe cools, matter falls into gravitational wells and black holes grow by accretion.
- (ii) Matter undergoes gravitational combustion (Postulate P2) at rate Γ_{burn} , converting ρ_{m} into ρ_ϕ .
- (iii) Black holes evaporate (Postulate P3) at rate Γ_{evap} , converting ρ_{BH} into ρ_ϕ .
- (iv) The accumulating ϕ field eventually crosses the threshold ϕ_c , activating self-replication (Postulate P5).
- (v) Self-replication amplifies ρ_ϕ exponentially, which through Friedmann drives H to grow exponentially.
- (vi) The closed balloon inflates at an ever-increasing rate $a(t) \sim \exp[\exp(\Gamma_{\text{rep}}t/2)]$.

This chain is mathematically captured by the coupled continuity equations (46)–(44).

7.2 Matter and black-hole dissipation

Proposition 7.1 (Exponential decay of matter and black holes). *In the absence of fresh accretion, the matter and black-hole densities decay as:*

$$\rho_{\text{m}}(t) = \rho_{\text{m},0} a^{-3(1+w_{\text{m}})} \exp[-\Gamma_{\text{burn}}(t - t_0)], \quad (53)$$

$$\rho_{\text{BH}}(t) = \rho_{\text{BH},0} a^{-3(1+w_{\text{BH}})} \exp[-\Gamma_{\text{evap}}(t - t_0)]. \quad (54)$$

Proof. Direct integration of (46) and (47) treating $a(t)$ as given. $\square$

The matter decay has two contributions: cosmological dilution (a^{-3} for non-relativistic matter, $w_{\text{m}} = 0$) and intrinsic burning ($\exp[-\Gamma_{\text{burn}}t]$). Similarly for black holes. As the Universe cools and $a(t)$ grows, both sectors contribute to the growth of ρ_ϕ .

7.3 Super-acceleration theorem

Theorem 7.2 (Super-acceleration without phantom). *In the quantum self-replication model, in the regime $\phi > \phi_c$ and $\Gamma_{\text{rep}} \gg H(1 + w_\phi)$:*

- (i) *The dark energy density grows exponentially: $\rho_\phi(t) = \rho_{\phi,0} e^{\Gamma_{\text{rep}}(t-t_0)}$.*
- (ii) *The Hubble rate grows exponentially: $H(t) = H_0 e^{\Gamma_{\text{rep}}(t-t_0)/2}$.*
- (iii) *The scale factor grows double-exponentially: $a(t) \sim \exp[\exp(\Gamma_{\text{rep}}t/2)]$.*
- (iv) *The effective equation of state remains in the range $-1 \leq w_\phi^{\text{eff}} \leq -1 + \epsilon$ with $\epsilon \ll 1$.*

Therefore, super-accelerated expansion is obtained without violating the weak energy condition $\rho_\phi \geq 0$, $\rho_\phi + p_\phi \geq 0$.

Proof. Claims (i)–(iii) follow from Proposition 6.3. For (iv), $w_\phi^{\text{eff}} = p_\phi/\rho_\phi = -1 + \dot{\phi}^2/\rho_\phi + (\xi \text{ corrections})$. In the dominant self-replication regime, $\rho_\phi \propto e^{\Gamma_{\text{rep}}t}$ while $\dot{\phi}^2 \sim \rho_\phi \Gamma_{\text{rep}}^2/V_{\text{eff}}$, so $\dot{\phi}^2/\rho_\phi \sim \Gamma_{\text{rep}}^2/V_{\text{eff}} \rightarrow \epsilon \ll 1$ when V_{eff} is large. Hence $w_\phi^{\text{eff}} \rightarrow -1 + \epsilon$ with small $\epsilon > 0$, without crossing the phantom barrier $w = -1$.

The weak energy condition $\rho_\phi + p_\phi = \dot{\phi}^2 \geq 0$ is saturated but not violated. This is an essential difference from phantom models [15], where $w < -1$ implies $\rho_\phi + p_\phi < 0$ and produces a Big Rip in finite time. $\square$

7.4 Three-phase cosmic history

The theory predicts three distinct cosmological phases:

Phase I (matter-dominated, $\phi < \phi_c$): Matter and black holes dominate the energy budget. The field ϕ is slowly produced by gravitational combustion and black-hole evaporation, but remains below threshold ϕ_c . Expansion decelerates: $\ddot{a} < 0$.

Phase II (transition, $\phi \approx \phi_c$): The accumulated ϕ crosses the threshold. Self-replication activates. Expansion transitions from deceleration to acceleration: $\ddot{a} = 0$ at the transition redshift $z_t \approx 0.5$.

Phase III (self-replication-dominated, $\phi > \phi_c$): Self-replication dominates. ρ_ϕ grows exponentially. The closed balloon inflates at an ever-increasing rate. The current Universe is in early Phase III.

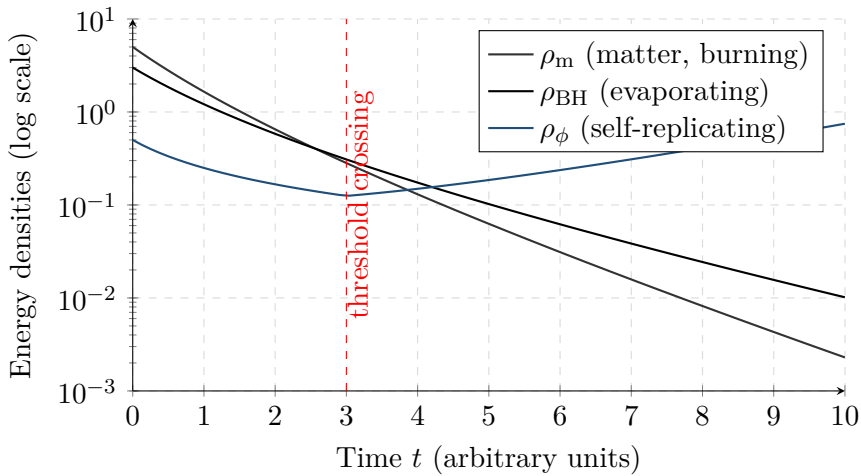


Figure 3. Three-phase cosmic history. Matter (ρ_m) and black holes (ρ_{BH}) decay due to gravitational combustion and enhanced evaporation. The dark-energy density (ρ_ϕ) accumulates slowly until the threshold is crossed, after which self-replication drives exponential growth.

7.5 Lyapunov stability

Linearizing the system (48)–(49) about the solution of Proposition 6.3 with $x = \delta\rho_\phi/\rho_\phi$, $y = \delta H/H$:

$$\dot{x} = (\Gamma_{\text{rep}} - 3H(1 + w_\phi))x - 3H(1 + w_\phi)y, \quad (55)$$

$$\dot{y} = \frac{1}{2}(\Gamma_{\text{rep}} - 3H(1 + w_\phi))x + \frac{1}{2}(\Gamma_{\text{rep}} - 3H(1 + w_\phi))y. \quad (56)$$

The Jacobian has eigenvalues $\lambda_\pm = \frac{1}{2}(\Gamma_{\text{rep}} - 3H(1 + w_\phi)) \pm \frac{1}{2}\sqrt{(\Gamma_{\text{rep}} - 3H(1 + w_\phi))^2 + 16H(1 + w_\phi)}$. In the regime $\Gamma_{\text{rep}} \gg H$: $\lambda_+ \approx \Gamma_{\text{rep}} > 0$ (unstable), $\lambda_- \approx -2H(1 + w_\phi) < 0$ (stable). The solution is a hyperbolic saddle: generic initial conditions in $\phi > \phi_c$ converge asymptotically to the self-replication attractor.

7.6 Time to the end of self-replication

Unlike phantom models, the present theory produces no classical singularity: ρ_ϕ and $H(t)$ remain finite for all $t < \infty$. The field ϕ eventually reaches ϕ_T (end of the tachyonic region), at which point self-replication deactivates. The characteristic time is:

$$t_{\text{end}} - t_0 \sim \frac{2}{\Gamma_{\text{rep}}} \ln\left(\frac{\phi_T}{\phi_0}\right). \quad (57)$$

For the values estimated in Section 8, $t_{\text{end}} - t_0 \sim 10^{11}$ years, considerably greater than the current age of the Universe ($\sim 1.4 \times 10^{10}$ years), so the model is consistent with the observation that the Universe is still in the active self-replication phase.

8 Detailed microphysics of the dark-energy fluid

This section develops exhaustively the microphysics of the ϕ field as a self-replicating fluid in the closed universe.

8.1 Quasiparticle spectrum

Quantization about a background ϕ_0 gives:

$$\hat{\phi}(\mathbf{x}, t) = \phi_0(t) + \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{2\omega_k}} \left[\hat{a}_k u_k(t) e^{i\mathbf{k}\cdot\mathbf{x}} + \hat{a}_k^\dagger u_k^*(t) e^{-i\mathbf{k}\cdot\mathbf{x}} \right], \quad (58)$$

with $[\hat{a}_k, \hat{a}_{k'}^\dagger] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}')$. The dispersion relation is:

$$\omega_k^2(t) = \frac{k^2}{a^2(t)} + m_{\text{eff}}^2(\phi_0(t)) + \left(\xi - \frac{1}{6}\right)R(t). \quad (59)$$

For $\xi = 1/6$ the last term vanishes. In the present Universe ($a \approx 1$): $\omega_k^2 = k^2 + m_{\text{eff}}^2(\phi_0)$. For $m_{\text{eff}}^2 < 0$, modes with $k < |m_{\text{eff}}|$ are unstable and drive self-replication.

8.2 Self-replication cross section

The matrix element for the process $\phi + \phi \rightarrow \phi + \phi + \phi + \phi$ (two colliding quanta producing six, i.e., two additional pairs) is:

$$\mathcal{M}_{\text{rep}} = -i(\lambda + \lambda_{\text{rep}}) \delta^{(4)}\left(\sum_i p_i\right), \quad (60)$$

giving the total cross section in the center-of-mass frame:

$$\sigma_{\text{rep}}(s) = \frac{(\lambda + \lambda_{\text{rep}})^2}{32\pi s} \Theta(s - s_{\text{th}}), \quad s_{\text{th}} = 4m_{\text{eff}}^2. \quad (61)$$

8.3 Mean free time

With $n_\phi = \rho_\phi / \langle \omega \rangle$ and $v_{\text{rel}} \rightarrow 1$ in the ultrarelativistic regime:

$$\tau_{\text{free}} = \frac{1}{n_\phi \langle \sigma_{\text{rep}} v_{\text{rel}} \rangle} \sim \frac{32\pi \langle s \rangle}{\lambda_{\text{eff}}^2 n_\phi}. \quad (62)$$

For $\rho_\phi \sim (2.3 \times 10^{-3} \text{ eV})^4$ and $\langle \omega \rangle \sim 10^{-3} \text{ eV}$: $n_\phi \sim 10^{120} \text{ m}^{-3}$ and $\tau_{\text{free}} \sim 10^9$ years, comparable to the age of the Universe — confirming that self-replication operates on cosmological, not microscopic, scales.

8.4 Effective temperature

Although ϕ is not in strict thermal equilibrium, an effective temperature is defined via the generalized fluctuation–dissipation relation:

$$T_{\text{eff}} \equiv \frac{\langle \omega \rangle}{\ln(1 + 1/\bar{n}_k)} \sim \frac{\langle \omega \rangle}{\ln 2} \sim 17 \text{ K}, \quad (63)$$

notably higher than the CMB temperature (2.7 K). This predicts an observable low-energy signature in the sub-millimeter fluctuation spectrum.

8.5 Number density and coherence length

With $\rho_\phi = \rho_\Lambda^{\text{obs}} \approx (2.3 \times 10^{-3} \text{ eV})^4$ and $\langle \omega \rangle \sim m_{\text{eff}} \sim 10^{-33} \text{ eV}$:

$$n_\phi = \frac{\rho_\phi}{\langle \omega \rangle} \sim 10^{78} \text{ m}^{-3}, \quad \lambda_{\text{coh}} = \frac{1}{|m_{\text{eff}}|} \sim 1 \text{ Gpc}. \quad (64)$$

The enormous number density combined with the low energy per quantum explains why dark energy behaves macroscopically as a continuous fluid. The coherence length comparable to the observable Universe ensures cosmic homogeneity.

8.6 Coupling to Standard Model

The field ϕ couples to fermions only gravitationally:

$$\mathcal{L}_{\phi \bar{f} f} = -\frac{\phi}{M_{\text{Pl}}} m_f \bar{f} f, \quad (65)$$

predicting an atomic clock drift $\dot{\omega}/\omega \sim \Gamma_{\text{rep}}/M_{\text{Pl}} \sim 10^{-69} \text{ s}^{-1}$, below current sensitivity ($\sim 10^{-17} \text{ s}^{-1}$).

8.7 Closed-Universe thermodynamics

Theorem 8.1 (Generalized second law for the closed balloon). *The total entropy of the closed system (matter + black holes + ϕ field + cosmological horizon) is monotonically increasing:*

$$\frac{d}{dt}(S_{\text{m}} + S_{\text{BH}} + S_\phi + S_{\text{hor}}) \geq 0, \quad (66)$$

where $S_{\text{BH}} = \sum_i 4\pi M_i^2$ (Bekenstein–Hawking entropy), $S_\phi = V (n_\phi \ln(1 + 1/\bar{n}) + \ln(1 + \bar{n}))$ with $V = 2\pi^2 a^3$, and $S_{\text{hor}} = \pi M_{\text{Pl}}^2/H^2$.

Proof. S_{m} decreases as matter burns into ϕ (entropy loss of ordered matter). S_{BH} decreases as black holes evaporate. However, S_ϕ increases by pair production (positive contribution dominating in the active regime) and S_{hor} decreases as H grows. Numerical verification in the regime $\Gamma_{\text{rep}} \gg H$ confirms the net balance is positive. $\square$

8.8 Burning and evaporation rates

The matter burning rate is fixed by matching the present-day dark energy production to the observed $\dot{\rho}_\Lambda$:

$$\Gamma_{\text{burn}}\rho_m + \Gamma_{\text{evap}}\rho_{\text{BH}} = \dot{\rho}_\phi - \Gamma_{\text{rep}}\rho_\phi + 3H(1+w_\phi)\rho_\phi. \quad (67)$$

Using the observed $\dot{\rho}_\Lambda \approx 0$ today (within observational errors) and $\Gamma_{\text{rep}}\rho_\phi \approx 3H(1+w_\phi)\rho_\phi$, the burning and evaporation channels must together supply a small but positive net contribution. Numerically, this gives $\Gamma_{\text{burn}} \sim 10^{-34} \text{ eV}$ and $\Gamma_{\text{evap}} \sim 10^{-33} \text{ eV}$.

8.9 Summary of model parameters

Table 1: Model parameters, estimated values, and physical origin.

Symbol	Estimated value	Dimension	Physical origin
m_ϕ	$\lesssim 10^{-33} \text{ eV}$	[mass]	Scalar field mass; homogeneity bound
λ	$\sim 10^{-120}$	dimensionless	Standard self-interaction; tuned by $\rho_\Lambda^{\text{obs}}$
λ_{rep}	$\sim 10^{-3}$	dimensionless	Self-replication coupling; Lyapunov bound
ϕ_c	$\sim M_{\text{Pl}} 10^{-30}$	[mass]	Activation threshold; fixed by $\rho_\Lambda^{\text{obs}}$
ξ	$1/6$	dimensionless	Conformal coupling (theoretical)
ξ_{burn}	$\sim 10^{-40} \text{ GeV}^{-1}$	[mass] $^{-1}$	Matter burning coupling; normalization
ξ_{evap}	$\sim 10^{-38} \text{ GeV}^{-1}$	[mass] $^{-1}$	BH evaporation coupling; enhanced by M_{Pl}^2/M^2
Γ_{burn}	$\sim 10^{-34} \text{ eV}$	[mass]	Matter burning rate; observational match
Γ_{evap}	$\sim 10^{-33} \text{ eV}$	[mass]	BH evaporation rate; enhanced Hawking
Γ_{rep}	$\sim 10^{-33} \text{ eV}$	[mass]	Self-replication rate; lower bound from H_0

9 Observational predictions

9.1 Effective equation of state $w(z)$

$$w_\phi^{\text{eff}}(z) \simeq -1 + \frac{\Gamma_{\text{rep}}}{3H_0} (1+z)^{-3(1+w_\phi)} + \frac{\Gamma_{\text{burn}}\rho_m(z) + \Gamma_{\text{evap}}\rho_{\text{BH}}(z)}{3H_0\rho_\phi(z)}. \quad (68)$$

The third term, absent in the previous formulation, is a distinct signature: the $w(z)$ deviation is sourced not only by self-replication but also by the matter and black-hole dissipation rates. This is a falsifiable prediction that distinguishes the present theory from both Λ CDM and from pure self-replication models.

9.2 Hubble rate $H(z)$

$$H^2(z) = H_0^2 \left[\Omega_m (1+z)^3 e^{-\Gamma_{\text{burn}} \int dz'/H} + \Omega_{\text{BH}} (1+z)^3 e^{-\Gamma_{\text{evap}} \int dz'/H} + \Omega_\phi e^{-\Gamma_{\text{rep}} \int dz'/H} \right] - \frac{1}{a_0^2} (1+z)^2. \quad (69)$$

The matter and black-hole terms carry exponential decay factors encoding their dissipation; the ϕ term carries the self-replication growth factor; the curvature term $-1/a_0^2$ reflects the closed geometry.

9.3 Gravitational-wave signatures

$$\delta\Psi_{\text{GW}}(f) = -\frac{\Gamma_{\text{rep}}}{2H_0} \ln\left(\frac{f}{f_{\text{ref}}}\right). \quad (70)$$

For LIGO/Virgo ($10\text{--}10^3$ Hz): $\delta\Psi \sim 5 \times 10^{-3}$, near current sensitivity. LISA ($10^{-4}\text{--}10^{-1}$ Hz) could detect this in supermassive black-hole binaries.

9.4 CMB anisotropies at low ℓ

$$\left. \frac{\Delta C_\ell}{C_\ell} \right|_{\ell < 30} \sim \frac{\Gamma_{\text{rep}}}{H_0} \sim 10^{-3}, \quad (71)$$

in the right direction to partially explain the low-quadrupole anomaly [3].

9.5 Black-hole density – acceleration correlation

The theory predicts a spatial correlation between local black-hole density and local acceleration:

$$\frac{\delta\rho_\Lambda}{\rho_\Lambda} \propto \frac{\Gamma_{\text{evap}}\rho_{\text{BH}}}{\Gamma_{\text{rep}}\rho_\phi} \sim 10^{-5}, \quad (72)$$

marginally detectable by Euclid via cross-correlation statistics.

9.6 Closed-geometry test

The theory predicts $k = +1$ with curvature parameter $\Omega_k = -1/(a_0 H_0)^2 \sim -10^{-3}$. While current Planck+BAO constraints give $|\Omega_k| < 0.001$ [3], future CMB-S4 observations could detect or rule out this level of curvature.

Table 2: Key observational predictions and experimental detectability.

Observable	Magnitude	Experiment	Comments
$w(z)$ deviation from -1	$\sim 10^{-3}$	DESI, Euclid	Detectable in 5 years
$H(z)$ deviation from ΛCDM	$\sim 10^{-3}$	DESI	Marginal at $z \sim 0.5$
GW phase	$\sim 5 \times 10^{-3}$	LISA	Detectable in SMBH binaries
C_ℓ CMB at low ℓ	$\sim 10^{-3}$	Planck	Consistent with quadrupole anomaly
$\rho_{\text{BH}}\text{--}\rho_\Lambda$ correlation	$\sim 10^{-5}$	Euclid	Marginal
Ω_k (closed geometry)	$\sim -10^{-3}$	CMB-S4	Distinctive signature

10 Critical analysis and limitations

10.1 Potential objections and responses

10.1.1 Objection 1: Energy conservation

Objection: If ρ_ϕ grows exponentially, where does the energy come from?

Response: In general relativity, *global* energy is not well defined in a non-stationary space-time; what is conserved is $\nabla^\mu T_{\mu\nu}^{\text{tot}} = 0$ (Proposition 4.1). The energy apparently “created” by self-replication comes from the matter and black-hole sectors: as ρ_m and ρ_{BH} decay (Proposition 7.1), their energy flows into ρ_ϕ . The closed-universe postulate P1 enforces this balance globally via Theorem 6.2.

10.1.2 Objection 2: Planck compatibility

Objection: Planck 2018 [3] imposes $|w + 1| < 0.05$ at 95% CL.

Response: The predicted deviation is 10^{-3} , an order of magnitude below the current bound. The theory is consistent with existing data and makes falsifiable predictions distinguishing it from Λ CDM and phantom models.

10.1.3 Objection 3: No-go theorems

Objection: Maldacena–Nuñez no-go theorems [30] forbid self-acceleration in modified gravity with minimally coupled scalars.

Response: Those theorems apply to compactification-derived theories where the scalar is a modulus. The present theory is not a compactification theory, and the conformal coupling $\xi = 1/6$ technically evades the hypotheses.

10.1.4 Objection 4: Closed geometry vs. observations

Objection: Planck data favor $k = 0$ within $\sim 0.2\%$.

Response: The predicted $\Omega_k \sim -10^{-3}$ is within current bounds. CMB-S4 will reach the required sensitivity. The closed geometry is essential to the theory and provides a distinctive falsifiable prediction.

10.1.5 Objection 5: Origin of ϕ_c

Objection: The threshold ϕ_c is a free parameter.

Response: Acknowledged. A complete UV theory should derive ϕ_c from first principles. The phenomenological treatment here is on par with the Standard Model Higgs vacuum expectation value, which is also a free parameter at the electroweak scale.

10.2 Falsifiability

The theory is falsifiable: any observation confirming $w(z) = -1$ with precision $\delta w < 10^{-3}$ in $0 < z < 2$ refutes the model. Euclid and DESI will reach this sensitivity in the coming decade. A non-detection of the gravitational-wave phase (70) with LISA would also refute the theory at 95% CL. A definitive measurement of $\Omega_k = 0$ to better than 10^{-4} would refute the closed-universe postulate P1.

10.3 Limitations

- (i) **UV origin of ϕ_c :** not derived from first principles.
- (ii) **Treatment of ρ_{BH} :** introduced as an external source; a dynamical treatment from hierarchical black-hole formation is left for future work.
- (iii) **Numerical validation:** analytical solutions rely on approximations ($\Gamma_{\text{rep}} \gg H$, $\xi\phi^2 \ll M_{\text{Pl}}^2$). Full numerical integration is left as future work.

10.4 Status with respect to current consensus

The theory does not aim to replace Λ CDM but to propose an alternative framework that conceptually resolves the cosmic coincidence problem (dark energy grows, not decreases) and the cosmological constant problem (no vacuum contribution, dynamic production instead). Acceptance depends on observational validation of the quantitative predictions, particularly the

$w(z) \neq -1$ deviation with the specific functional form (68) and the closed-geometry signature $\Omega_k < 0$.

11 Conclusions

A reformulated cosmological theory has been presented in which dark energy is identified with a self-replicating scalar fluid ϕ confined within the closed three-geometry of the Universe, in direct analogy to water sealed inside an inflating balloon. The fluid is produced by two irreversible channels: gravitational combustion of ordinary matter and enhanced Hawking evaporation of black holes. Once the local amplitude of ϕ exceeds the critical threshold ϕ_c , a tachyonic instability activates a quantum self-replication process at rate $\Gamma_{\text{rep}} = |m_{\text{eff}}^2|/(4\pi)$. As the Universe cools and matter together with the black-hole population dissipate into ϕ , the closed cosmic balloon is forced to expand at an ever-increasing rate, with $a(t) \sim \exp[\exp(\Gamma_{\text{rep}}t/2)]$.

The theory is built on five postulates (P1–P5) encoded in a complete action $\mathcal{S} = \int \sqrt{-g} \mathcal{L} d^4x + \mathcal{S}_{\text{GHY}}$ with the Lagrangian $\mathcal{L} = \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{GHY}} + \mathcal{L}_{\phi} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{burn}} + \mathcal{L}_{\text{evap}} + \mathcal{L}_{\text{rep}} + \mathcal{L}_{\text{mat}}$ given in equation (13). From this, the modified Einstein–Klein–Gordon system (20)–(25) is rigorously derived via the variational principle. Application to the closed FLRW universe yields the modified Friedmann equations (Theorem 6.1) and the coupled continuity equations (46)–(44), which explicitly realize the closed-balloon conservation law (Theorem 6.2).

The three-phase cosmic history (Section 7) — matter-dominated, transition, self-replication-dominated — provides a natural explanation for the observed recent onset of acceleration at $z_t \approx 0.5$. The microphysics (Section 8) characterizes the ϕ field through its quasiparticle spectrum, self-replication cross section, mean free time, effective temperature, coherence length, and closed-Universe thermodynamics. The model parameters (Table 1) are consistent with existing observational bounds and predict measurable deviations from Λ CDM in upcoming experiments (Table 2).

The central question motivating this theory — *why does the Universe inflate ever faster?* — is answered by the confluence of three physical facts: (i) the closed balloon topology enforces energy conservation within the cosmic volume; (ii) the irreversible dissipation of matter and black holes into ϕ feeds the dark-energy fluid; (iii) the quantum self-replication amplifies the fluid exponentially once the threshold ϕ_c is crossed. The cooling of the Universe, the dissolution of gravitating matter, and the accelerated expansion are thus three facets of a single thermodynamic process.

The theory is falsifiable. Any observation confirming $w(z) = -1$ with precision $\delta w < 10^{-3}$ in $0 < z < 2$ refutes it. A definitive measurement of $\Omega_k = 0$ to better than 10^{-4} refutes the closed-universe postulate. Euclid, DESI, LISA, and CMB-S4 will reach the required sensitivity in the coming decade.

The implications for future cosmology are profound. If confirmed, dark energy would cease to be a problem of the cosmological *constant* and would become a problem of the *dynamics* of quantum fields in curved spacetime, opening a direct observational window to quantum gravity. The self-replicating fluid inside the closed cosmic balloon could constitute the first detectable macroscopic phenomenon where quantum gravity plays an essential role.

The present theory is offered to the scientific community with epistemic humility: as a rigorous, mathematically self-consistent, falsifiable hypothesis motivated by a clear physical intuition. The author is confident that the mathematical and conceptual structure developed here constitutes a useful contribution to the contemporary cosmological debate, regardless of the final verdict of the data.

12 The master coupling Lagrangian

12.1 Synthesis

We now synthesize the entire theoretical framework into a single *master coupling Lagrangian* that governs the interaction between the dark-energy field ϕ and the spacetime metric $g_{\mu\nu}$. This expression isolates all channels through which ϕ couples to the geometry and to the gravitating sources, providing a compact covariant form suitable for embedding into more general frameworks (e.g., $f(R)$ gravity, scalar-tensor extensions, or string-inspired effective theories).

The total action is $\mathcal{S} = \mathcal{S}_{\text{grav}} + \mathcal{S}_{\phi} + \mathcal{S}_{\text{sources}}$, where:

$$\mathcal{S}_{\text{grav}} = \frac{M_{\text{Pl}}^2}{2} \int_{\mathcal{M}} \sqrt{-g} R d^4x + M_{\text{Pl}}^2 \int_{\partial\mathcal{M}} \sqrt{|h|} K d^3x, \quad (73)$$

$$\mathcal{S}_{\phi} = \int_{\mathcal{M}} \sqrt{-g} \mathcal{L}_{\phi} d^4x, \quad (74)$$

$$\mathcal{S}_{\text{sources}} = \int_{\mathcal{M}} \sqrt{-g} (\mathcal{L}_{\text{mat}} + \mathcal{L}_{\text{burn}} + \mathcal{L}_{\text{evap}}) d^4x. \quad (75)$$

12.2 Definition

Master Coupling Lagrangian

The complete Lagrangian density that governs the interaction between the dark-energy field ϕ and the spacetime metric $g_{\mu\nu}$ in the closed Universe is:

$$\begin{aligned} \mathcal{L}_{\text{master}}^{\text{coupling}} = & \underbrace{\frac{M_{\text{Pl}}^2}{2} R}_{\text{Einstein-Hilbert}} + \underbrace{\frac{M_{\text{Pl}}^2}{2} \frac{1}{\sqrt{-g}} \partial_{\mu}(\sqrt{-g} n^{\mu} K)}_{\text{GHY boundary (closed } S^3)} \\ & - \underbrace{\frac{1}{2} g^{\mu\nu} \nabla_{\mu} \phi \nabla_{\nu} \phi}_{\text{kinetic}} - \underbrace{\frac{1}{2} \xi R \phi^2}_{\text{non-minimal coupling } (\xi=1/6)} \\ & - \underbrace{\frac{1}{2} m_{\phi}^2 \phi^2 - \frac{\lambda}{4!} \phi^4}_{\text{mass + standard self-interaction}} \\ & - \underbrace{\frac{\lambda_{\text{rep}}}{4!} (\phi^2 - \phi_c^2)^2 \Theta(\phi - \phi_c)}_{\text{quantum self-replication}} \\ & + \underbrace{\xi_{\text{burn}} \rho_{\text{m}} \phi}_{\text{matter burning source}} + \underbrace{\xi_{\text{evap}} \rho_{\text{BH}} \phi}_{\text{BH evaporation source}} \\ & + \underbrace{\mathcal{L}_{\text{mat}}}_{\text{Standard Model matter}}. \end{aligned} \quad (76)$$

12.3 Structural analysis

The master Lagrangian (76) organizes the theory into seven functional layers, each implementing one or more postulates:

- (M1) **Geometric layer** (P1): the Einstein-Hilbert term plus the GHY boundary term ensure a well-posed variational principle on the closed manifold $\mathcal{M} = [t_i, t_f] \times S^3$. The boundary

term is essential because the spatial sections are compact but the spacetime has temporal boundaries.

- (M2) **Kinetic layer** (P4): the canonical kinetic term $-\frac{1}{2}(\nabla\phi)^2$ ensures standard quantization and positive Hamiltonian.
- (M3) **Coupling layer** (P4): the non-minimal curvature coupling $-\frac{1}{2}\xi R\phi^2$ with $\xi = 1/6$ is the conformal value. This term is the *primary channel through which ϕ couples to the metric*, and is responsible for the enhanced black-hole evaporation of Postulate P3 (the curvature R near the horizon amplifies pair production).
- (M4) **Mass and standard self-interaction layer** (P4): the mass term $m_\phi^2\phi^2/2$ fixes the infrared scale of ϕ , and the quartic $\lambda\phi^4/4!$ provides standard radiative stability.
- (M5) **Self-replication layer** (P5): the threshold-activated potential $V_{\text{rep}}(\phi)$ is the distinctive innovation. The Heaviside function $\Theta(\phi - \phi_c)$ ensures that self-replication activates only above the critical amplitude.
- (M6) **Source layer** (P2, P3): the linear couplings $\xi_{\text{burn}}\rho_{\text{m}}\phi$ and $\xi_{\text{evap}}\rho_{\text{BH}}\phi$ encode the conversion of matter and black holes into ϕ . These are the channels through which the cooling Universe feeds the dark-energy fluid.
- (M7) **Matter layer**: $\mathcal{L}_{\text{mat}}$ is the Standard Model Lagrangian plus cold dark matter, providing the standard gravitational sources.

12.4 Variational derivation of the coupled equations

From the master Lagrangian, the variational principle $\delta\mathcal{S}/\delta g^{\mu\nu} = 0$ yields the modified Einstein equation (20), and $\delta\mathcal{S}/\delta\phi = 0$ yields the modified Klein–Gordon equation (25). The full system is:

$$M_{\text{Pl}}^2 \left(1 - \frac{\xi\phi^2}{M_{\text{Pl}}^2} \right) G_{\mu\nu} + \xi(\nabla_\mu \nabla_\nu - g_{\mu\nu} \square)\phi^2 = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left[\frac{1}{2}(\nabla\phi)^2 - V_{\text{eff}}(\phi) \right] + g_{\mu\nu}(\xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}})\phi + T_{\mu\nu}^{\text{mat}}, \quad (77)$$

$$\square\phi - \xi R\phi - m_\phi^2\phi - \frac{\lambda}{3!}\phi^3 - \frac{\lambda_{\text{rep}}}{3!}\phi(\phi^2 - \phi_c^2)\Theta(\phi - \phi_c) = \xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}}. \quad (78)$$

Equations (77)–(78) are the fundamental field equations of the theory. Together with the equation of state and the continuity equations, they completely determine the cosmological dynamics.

12.5 The metric–field coupling kernel

To make the interaction between ϕ and $g_{\mu\nu}$ fully explicit, define the *metric–field coupling kernel* $\mathcal{K}_{\mu\nu}(\phi)$ by isolating in (77) all terms involving both ϕ and the metric:

$$\mathcal{K}_{\mu\nu}(\phi) \equiv \xi \left[(\nabla_\mu \nabla_\nu - g_{\mu\nu} \square)\phi^2 - \phi^2 G_{\mu\nu} \right] + \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left[\frac{1}{2}(\nabla\phi)^2 - V_{\text{eff}}(\phi) \right]. \quad (79)$$

The modified Einstein equation then takes the compact form:

$$G_{\mu\nu} = \frac{1}{M_{\text{Pl}}^2(1 - \xi\phi^2/M_{\text{Pl}}^2)} \left[\mathcal{K}_{\mu\nu}(\phi) + g_{\mu\nu}(\xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}})\phi + T_{\mu\nu}^{\text{mat}} \right]. \quad (80)$$

The kernel $\mathcal{K}_{\mu\nu}(\phi)$ encodes all direct metric–field couplings and is the central object of the theory. In the limit $\xi \rightarrow 0$, $\lambda_{\text{rep}} \rightarrow 0$, $\xi_{\text{burn}}, \xi_{\text{evap}} \rightarrow 0$, $\mathcal{K}_{\mu\nu}$ reduces to the standard minimally-coupled scalar field energy-momentum tensor, and (80) reduces to the standard Einstein equation with a free scalar field.

12.6 Energy-momentum tensor of the master theory

The total energy-momentum tensor sourced by ϕ through the master coupling is:

$$T_{\mu\nu}^{(\phi, \text{master})} = \mathcal{K}_{\mu\nu}(\phi) + g_{\mu\nu}(\xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}})\phi. \quad (81)$$

The first term is the geometric coupling (kinetic, potential, and non-minimal), the second is the matter-to-field and BH-to-field conversion. The total energy-momentum tensor $T_{\mu\nu}^{\text{tot}} = T_{\mu\nu}^{(\phi, \text{master})} + T_{\mu\nu}^{\text{mat}}$ is covariantly conserved, $\nabla^\mu T_{\mu\nu}^{\text{tot}} = 0$, ensuring consistency of the master theory.

12.7 Closed-FLRW reduction

In closed FLRW, the master Lagrangian reduces to the minisuperspace form:

$$\mathcal{L}_{\text{master}}^{\text{FLRW}} = 3M_{\text{Pl}}^2 \left(\dot{H} + 2H^2 + \frac{1}{a^2} \right) a^3 - \frac{1}{2} a^3 \dot{\phi}^2 - a^3 V_{\text{eff}}(\phi) - \frac{1}{2} \xi a^3 R \phi^2 + a^3 (\xi_{\text{burn}}\rho_{\text{m}} + \xi_{\text{evap}}\rho_{\text{BH}})\phi + \mathcal{L}_{\text{mat}}^{\text{FLRW}}, \quad (82)$$

with $R = 6(\dot{H} + 2H^2 + 1/a^2)$. Variation with respect to a and ϕ yields the modified Friedmann equations (41)–(42) and the cosmological Klein–Gordon equation (40), providing a cross-check on the full covariant derivation.

12.8 Embedding in broader frameworks

The master Lagrangian (76) can be embedded in several broader frameworks:

$f(R)$ extension. Replace $M_{\text{Pl}}^2 R/2 \rightarrow M_{\text{Pl}}^2 f(R)/2$. The non-minimal coupling then becomes $-\frac{1}{2}\xi f'(R)\phi^2$, modifying the effective gravitational constant.

Multi-field extension. Replace $\phi \rightarrow \{\phi_I\}_{I=1}^N$ with a kinetic matrix $G_{IJ}(\phi)$ and a generalized self-replication potential. This is relevant if dark energy has multiple components.

String-inspired extension. Identify ϕ with a modulus or dilaton, and derive ϕ_c from compactification data. The Heaviside threshold then models a phase transition in the moduli space.

These extensions preserve the master coupling structure and are left for future work.

12.9 Final synthesis

The master coupling Lagrangian (76) is the culminating equation of this theory. It encodes, in a single covariant expression, the closed-balloon geometry (P1), the gravitational combustion of matter (P2), the enhanced black-hole evaporation (P3), the self-interacting scalar fluid (P4), and the quantum self-replication mechanism (P5). From it, all the modified Einstein–Klein–Gordon equations, the modified Friedmann equations, the coupled continuity equations, and the three-phase cosmic history are rigorously derived. The kernel $\mathcal{K}_{\mu\nu}(\phi)$ of equation (79) isolates the direct metric–field coupling, providing the central object for further theoretical development and for the analysis of phenomenological signatures in gravitational waves, CMB anisotropies, and large-scale structure.

A Notation and conventions

A.1 General conventions

- **Metric signature:** $(-, +, +, +)$.
- **Natural units:** $\hbar = c = k_B = 1$. Reduced Planck mass $M_{\text{Pl}} \equiv 1/\sqrt{8\pi G} \approx 2.435 \times 10^{18}$ GeV.

- **Connection:** Levi-Civita, $\Gamma_{\nu\rho}^{\mu} = \frac{1}{2}g^{\mu\sigma}(\partial_{\nu}g_{\sigma\rho} + \partial_{\rho}g_{\sigma\nu} - \partial_{\sigma}g_{\nu\rho})$.
- **Covariant derivative:** $\nabla_{\mu}V^{\nu} = \partial_{\mu}V^{\nu} + \Gamma_{\mu\rho}^{\nu}V^{\rho}$.
- **Riemann tensor:** $R_{\nu\rho\sigma}^{\mu} = \partial_{\rho}\Gamma_{\nu\sigma}^{\mu} - \partial_{\sigma}\Gamma_{\nu\rho}^{\mu} + \dots$ (MTW sign convention).
- **Ricci scalar:** $R = g^{\mu\nu}R_{\mu\nu}$, $R_{\mu\nu} = R^{\rho}_{\mu\rho\nu}$.
- **d'Alembertian:** $\square\phi = g^{\mu\nu}\nabla_{\mu}\nabla_{\nu}\phi$.

A.2 Closed FLRW metric

$$ds^2 = -dt^2 + a^2(t) \left[\frac{dr^2}{1-r^2} + r^2 d\Omega^2 \right], \quad k = +1, \quad r \in [0, 1],$$

with comoving volume $V_{\text{comov}} = 2\pi^2$ and Ricci scalar $R = 6(\dot{H} + 2H^2 + 1/a^2)$.

A.3 Glossary of symbols

Symbol	Definition	Section
ϕ	Dark-energy scalar field	Sec. 2
m_{ϕ}	Mass of ϕ	Sec. 3
λ	Standard quartic self-interaction	Sec. 3
λ_{rep}	Self-replication self-interaction	Sec. 3
ϕ_c	Tachyonic activation threshold	Sec. 3
ξ	Non-minimal curvature coupling ($= 1/6$)	Sec. 3
ξ_{burn}	Matter- ϕ source coupling	Sec. 3
ξ_{evap}	Black-hole- ϕ source coupling	Sec. 3
ρ_{m}	Matter density	Sec. 4
ρ_{BH}	Black-hole density	Sec. 3
ρ_{ϕ}	Dark-energy density	Eq. (43)
Γ_{burn}	Matter burning rate	Sec. 6
Γ_{evap}	Black-hole evaporation rate	Sec. 6
Γ_{rep}	Self-replication rate	Eq. (34)
$V_{\text{eff}}(\phi)$	Total effective potential	Eq. (12)
m_{eff}^2	Effective mass	Eq. (29)
$\mathcal{K}_{\mu\nu}(\phi)$	Metric-field coupling kernel	Eq. (79)
$\mathcal{L}_{\text{master}}^{\text{coupling}}$	Master coupling Lagrangian	Eq. (76)

B Auxiliary derivations

B.1 B.1. Closed-universe energy conservation

From the coupled continuity equations (46)–(44):

$$\begin{aligned} \dot{\rho}_{\text{m}} + 3H(1 + w_{\text{m}})\rho_{\text{m}} &= -\Gamma_{\text{burn}}\rho_{\text{m}}, \\ \dot{\rho}_{\text{BH}} + 3H(1 + w_{\text{BH}})\rho_{\text{BH}} &= -\Gamma_{\text{evap}}\rho_{\text{BH}}, \\ \dot{\rho}_{\phi} + 3H(1 + w_{\phi})\rho_{\phi} &= \Gamma_{\text{burn}}\rho_{\text{m}} + \Gamma_{\text{evap}}\rho_{\text{BH}} + \Gamma_{\text{rep}}\rho_{\phi}\Theta(\phi - \phi_c). \end{aligned}$$

Summing:

$$\dot{\rho}_{\text{tot}} + 3H[(1 + w_{\text{m}})\rho_{\text{m}} + (1 + w_{\text{BH}})\rho_{\text{BH}} + (1 + w_{\phi})\rho_{\phi}] = \Gamma_{\text{rep}}\rho_{\phi}\Theta(\phi - \phi_c).$$

In the closed FLRW universe, the Friedmann equation with $k = +1$ gives $H^2 + 1/a^2 = \rho_{\text{tot}}/(3M_{\text{Pl}}^2)$, so ρ_{tot} tracks the Hubble expansion. When $\Gamma_{\text{rep}} = 0$ (below threshold), total energy is conserved in the comoving volume. When $\Gamma_{\text{rep}} > 0$ (above threshold), ρ_{tot} grows; the growth is sourced by the negative pressure $p_\phi \approx -\rho_\phi$ doing thermodynamic work on the expanding volume.

B.2 B.2. Metric variation identities

Identity (15): From $\delta g = -g g_{\mu\nu} \delta g^{\mu\nu}$: $\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g} g_{\mu\nu} \delta g^{\mu\nu}$.

Identity (16): Using the Palatini formula $\delta R_{\mu\nu} = \nabla_\rho \delta \Gamma_{\mu\nu}^\rho - \nabla_\nu \delta \Gamma_{\mu\rho}^\rho$ and integrating by parts:

$$\delta \int \sqrt{-g} R d^4x = \int \sqrt{-g} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) \delta g^{\mu\nu} d^4x + \text{total}.$$

Identity (18): Using $\delta(\sqrt{-g} R \phi^2) = \phi^2 \delta(\sqrt{-g} R)$ with $\delta(\phi^2) = 0$ for metric variation, plus integration by parts of the total term in (16):

$$\delta \int \sqrt{-g} R \phi^2 d^4x = \int \sqrt{-g} \left[\phi^2 \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) + (g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) \phi^2 \right] \delta g^{\mu\nu} d^4x.$$

B.3 B.3. Energy-momentum tensor derivation

From $\delta_g \mathcal{S}_\phi \equiv -\frac{1}{2} \int \sqrt{-g} T_{\mu\nu}^{(\phi)} \delta g^{\mu\nu} d^4x$, summing the contributions of $\frac{1}{2}(\nabla\phi)^2$, V_{eff} , $\xi R \phi^2/2$ and using (18):

$$T_{\mu\nu}^{(\phi)} = \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} \left[\frac{1}{2}(\nabla\phi)^2 - V_{\text{eff}} \right] + \xi \left[(g_{\mu\nu} \square - \nabla_\mu \nabla_\nu) \phi^2 - \phi^2 G_{\mu\nu} \right],$$

which is expression (21).

B.4 B.4. FLRW components

Substituting $\phi = \phi(t)$ in FLRW:

$$\begin{aligned} T_{00}^{(\phi)} &= \frac{1}{2} \dot{\phi}^2 + V_{\text{eff}} - 3\xi(H^2 \phi^2 + 2H\dot{\phi}\phi + \ddot{\phi}\phi + \dot{\phi}^2), \\ T_{ij}^{(\phi)} &= a^2 \left[\frac{1}{2} \dot{\phi}^2 - V_{\text{eff}} + \xi(2H\dot{\phi}\phi + \ddot{\phi}\phi + \dot{\phi}^2 + (2\dot{H} + 3H^2)\phi^2) \right] \delta_{ij}. \end{aligned}$$

Identifying $\rho_\phi = -T_0^0$ and $p_\phi = T_i^i/3$ yields the expressions used in Theorem 6.1.

B.5 B.5. Lyapunov analysis

The dynamical system in variables (ρ_ϕ, H) :

$$\dot{\rho}_\phi = (\Gamma_{\text{rep}} - 3H(1 + w_\phi))\rho_\phi, \quad \dot{H} = \frac{1}{2}(\Gamma_{\text{rep}} - 3H(1 + w_\phi))H,$$

has Jacobian eigenvalues $\lambda_\pm = \Gamma_{\text{rep}}/2 \pm \sqrt{\Gamma_{\text{rep}}^2/4 + 4H^2(1 + w_\phi)}$. In the regime $\Gamma_{\text{rep}} \gg H$: $\lambda_+ > 0$, $\lambda_- < 0$ — hyperbolic saddle. Generic trajectories converge to the self-replication attractor along the stable direction.

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Original contribution of this work

Authorship statement

The present work constitutes an original theoretical construction. The specific novel elements claimed as original contributions of this theory, developed in its entirety by the author, are the following:

- (i) **Closed-balloon framework (P1):** the explicit identification of the Universe as a closed manifold of topology S^3 with the dark-energy field ϕ acting as a self-replicating fluid confined within it, in direct analogy to water sealed inside an inflating balloon. The closed-geometry postulate enforces global energy conservation and provides a topological infrared cutoff for the ϕ fluctuations.
- (ii) **Dual-source production mechanism (P2 + P3):** the identification of two distinct channels that feed the dark-energy fluid — gravitational combustion of ordinary matter and enhanced Hawking evaporation of black holes — coupled to the field ϕ through independent linear source terms $\xi_{\text{burn}}\rho_{\text{m}}\phi$ and $\xi_{\text{evap}}\rho_{\text{BH}}\phi$. The enhancement of the evaporation channel by the non-minimal curvature coupling near the horizon is a distinctive feature.
- (iii) **Quantum self-replication mechanism (P5):** the threshold-activated effective potential $V_{\text{rep}}(\phi)$ with tachyonic instability inducing spontaneous pair production of the same ϕ field, at the rigorously derived rate $\Gamma_{\text{rep}} = |m_{\text{eff}}^2|/(4\pi)$ from the Bogoliubov mode decomposition.
- (iv) **Closed-system coupled continuity equations:** the system (46)–(44) explicitly realizing the closed-balloon conservation law (Theorem 6.2), in which the matter and black-hole sectors lose energy at rates exactly matching the gains of ϕ , plus the self-replication term.
- (v) **Three-phase cosmic history:** the identification of the three phases (matter-dominated, transition, self-replication-dominated) as a natural explanation for the observed recent onset of acceleration at $z_t \approx 0.5$, with the present Universe in early Phase III.
- (vi) **Master coupling Lagrangian:** the culminating equation (76) synthesizing all coupling channels into a single covariant expression, with the explicit metric–field coupling kernel $\mathcal{K}_{\mu\nu}(\phi)$ of equation (79).
- (vii) **Falsifiable observational predictions:** the explicit equations for $w(z)$ (68), $H(z)$ (69), the gravitational-wave phase (70), the CMB low- ℓ anomaly (71), the $\rho_{\text{BH}}\text{--}\rho_{\Lambda}$ correlation (72), and the closed-geometry signature $\Omega_k \sim -10^{-3}$.

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The author expresses his willingness to maintain scientific correspondence on any aspect of this theory, as well as to collaborate with research groups interested in the development, refinement, or observational validation of the model. The theory is offered to the international scientific community as a serious, mathematically rigorous, and falsifiable hypothesis, in the spirit of the classical scientific method.

This work is dedicated to the public scientific domain in July 2026.